ROBUST ORDINAL REGRESSION

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INTRODUCTION

Making any type of decision, from buying a car to siting a nuclear plant, from choosing the best student deserving a scholarship to ranking the cities of the world according to their liveability, involves the evaluation of several alternatives with respect to different aspects, technically called *evaluation criteria*. Multiple Criteria Decision Aiding (MCDA) (see [1, 2]) provides methodologies to recommend the Decision Maker (DM) a decision that best fits the DM's preferences. Formally, in MCDA, a set of n alternatives $A = \{a_1, \ldots, a_n\}$ is evaluated with respect to a consistent family of m criteria $G = \{g_1, \ldots, g_m\}$ [3]. In general, each criterion $g_j \in G$ can be considered as a real-valued function $g_j : A \rightarrow \mathcal{I}_j \subseteq \mathbb{R}$, where the elements of $\mathcal{I}_j$ are real numbers having either the meaning of quantities for quantitative criteria or the meaning of ordered identifiers for qualitative criteria, for example, 1 = "bad", 2 = "medium", 3 = "good". Each criterion g_j can have an increasing or a decreasing direction of preference. In the first case, the higher the evaluation $g_j(a)$, the better a is with respect to criterion g_j; in the other case, the higher the evaluation $g_j(a)$, the worse a is with respect to criterion g_j. For example, evaluating a car involves both quantitative and qualitative criteria having increasing or decreasing direction of preference. Price and acceleration are typical quantitative criteria, whereas comfort and safety are qualitative criteria. Among these, acceleration, comfort and safety have an increasing direction of preference, whereas price has a decreasing direction of preference. In the following, without loss of generality, we shall suppose that criteria have increasing direction of preference.

According to Roy [4], in MCDA, the following three most important decision problems are considered:

- the *choice* problem, requiring to select a small number (as small as possible) of "good" alternatives in such a way that a single alternative may finally be chosen;
- the *sorting* problem, requiring to assign each alternative to some predefined and ordered categories;
- the *ranking* problem, requiring to define a complete or partial order on A; this preorder is the result of a procedure allowing to put together in classes alternatives that can be judged indifferent and to rank these classes.

Given two alternatives $a, b \in A$ and considering their evaluations on the m criteria belonging to family G, very often a will be better than b for some of the criteria, whereas b will be better than a for the remaining criteria. For this reason, in order to cope with the three above-mentioned problems, it

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is necessary to aggregate the evaluations of the alternatives, considering the preferences of the DM. In the literature, the three best known ways of aggregation are the following:

- assigning to each alternative $a \in A$ a real number synthesizing the evaluations of a on the m criteria and being representative of the desirability of a with respect to the problem at hand, as it is the case in MAUT—multiattribute utility theory [5, 6],
- building some outranking preference relation S on A, such that for any $a, b \in A$, aSb means that a is at least as good as b, as it is the case in outranking methods [3, 7, 8],
- using a set of "if..., then..." decision rules induced from the DM's preference information through dominance-based rough set approach (DRSA, see Refs 9–12).

The above-mentioned three ways of aggregation lead to three models of DM's preferences, called shortly *preference models*. They have been compared at an axiomatic level with respect to their capacity of representation. The comparison leads to a conclusion that the decision rule model is the most general and able to represent the most complex interactions among criteria [13, 14].

In the case of the first model, in order to assign a real number to each alternative, we consider a value function $U : \prod_{j=1}^{m} \mathcal{I}_j \to \mathbb{R}$, such that for any $a, b \in A$, a is at least as good as b ($a \succsim b$) if $U(g_1(a), \ldots, g_m(a)) \geq U(g_1(b), \ldots, g_m(b))$. The simplest form of the value function is the additive form, defined as $U(g_1(a), \ldots, g_m(a)) = \sum_{j=1}^{m} u_j(g_j(a))$, where $u_j(g_j(a))$ are nondecreasing functions of their arguments. In the following, for simplicity of notation, we shall use $U(a)$ instead of $U(g_1(a), \ldots, g_m(a))$ for all $a \in A$.

In the case of the second model, we consider a function $S : \prod_{j=1}^{m} \mathcal{I}_j \times \prod_{j=1}^{m} \mathcal{I}_j \to \mathbb{R}$, non-decreasing in its first m arguments and nonincreasing in its last m arguments, such that for each $a, b \in A$, we have

- $S((g_1(a), \ldots, g_m(a)), (g_1(b), \ldots, g_m(b))) = 1$ if aSb, and $S((g_1(a), \ldots, g_m(a)), (g_1(b), \ldots, g_m(b))) = 0$ otherwise, in case a crisp outranking relation is considered,
- a outranks b with a credibility $S((g_1(a), \ldots, g_m(a)), (g_1(b), \ldots, g_m(b))) \in [0, 1]$ in case a fuzzy outranking relation is considered.

In the case of the third model, starting from some preference information provided by the DM in the form of decision examples, the aim is to express relationships between the comprehensive decision concerning an alternative and its evaluations on relevant criteria, using "if..., then..." decision rules, such as

- "if maximum speed of a car is at least 175 km/h, and its price is at most 12,000 euro, then this car is comprehensively at least medium."

The choice of a multicriteria decision-aiding method well adapted to the decision context depends on several aspects of the decision process and the cooperation between the analyst and the DM [15].

ORDINAL REGRESSION

Each decision model requires specification of some parameters. For example, using MAUT, the parameters are related to the formulation of the marginal value functions $u_j(g_j(a))$, $j = 1, \ldots, m$. Eliciting direct preference information from the DM can be counterproductive in real-world decision-making situations because of a high cognitive effort required. Consequently, within MCDA, many methods have been proposed to determine the parameters characterizing the considered decision model in an indirect way, that is, inducing the values of such parameters from some holistic preference comparisons of alternatives given by the DM. This indirect preference elicitation is less demanding of cognitive effort and it is mainly used in the ordinal regression paradigm.

The most well-known ordinal regression methodology is the UTA (UTilités Additives) method proposed by Jacquet-Lagrèze and

Siskos [16], which aims at inferring one or more additive value functions from a given complete ranking on a reference set of alternatives A^R. The method considers a piecewise additive value function $U(a) = \sum_{j=1}^{m} u_j(g_j(a))$ having marginal value functions $u_j(\cdot), j = 1, \ldots, m$, being piecewise linear, with a predefined number of linear pieces. UTA uses linear programming to assess the additive value function compatible with the preference information provided by the DM. Technically, we have to solve a linear programming problem of the type:

$$\left.\begin{array}{l} \max\ \varepsilon,\ \text{s.t.} \\ \quad U(a^*) \geq U(b^*) + \varepsilon \quad \text{if} \quad a^* \succ b^*,\ \text{with}\ a^*, b^* \in A^R, \\ \quad U(a^*) = U(b^*) \quad \text{if} \quad a^* \sim b^*,\ \text{with}\ a^*, b^* \in A^R, \\ \left.\begin{array}{l} \sum_{j=1}^{m} u_j(\beta_j) = 1,\ \ u_j(\alpha_j) = 0,\ j = 1, \ldots, m, \\ u_j(g_j(a)) \geq u_j(g_j(b)) \quad \text{if} \quad g_j(a) \geq g_j(b), \forall a, b \in A,\ j = 1, \ldots, m, \end{array}\right\} E \end{array}\right\} E^A$$

where

- β_j and α_j are the best and the worst considered values of criterion $g_j, j = 1, \ldots, m$,
- $\succ$ and $\sim$ are the asymmetric and the symmetric part of the binary relation $\succsim$, representing the DM's preference information, that is, $a^* \succsim b^*$ means that a^* is at least as good as b^* for the DM,
- here, as always in the following, ε is considered without any constraint on the sign.

If the set of constraints E^A is feasible and $\varepsilon^* > 0$, then there exists at least one additive value function compatible with the DM's preferences. If there is no compatible value function, that is, if the preferences of the DM cannot be represented by an additive value function with pre-defined number of linear pieces, [16] suggests either to increase the number of linear pieces in some marginal value functions or to select the utility function U that gets the sum of deviation errors close to minimum and minimizes the number of ranking errors in the sense of Kendall or Spearman distance.

The ordinal regression paradigm has been applied within the two main MCDA approaches: those using a value function as preference model [16–20], and those using an outranking relation as preference model [21, 22].

ROBUST ORDINAL REGRESSION

Usually, from among many sets of parameters of a preference model representing the preference information given by the DM, only one specific set is selected and used to work out a recommendation.

As the selection of one from among many sets of parameters compatible with the preference information given by the DM is rather arbitrary, *Robust Ordinal Regression* (ROR) proposes taking into account all the sets of parameters compatible with the preference information, in order to give a recommendation in terms of necessary and possible consequences of applying all the compatible preference models on the considered set of alternatives: the *necessary* weak preference relation holds for any two alternatives $a, b \in A$ ($a \succsim^N b$) if and only if a is at least as good as b for all compatible preference models, whereas the *possible* weak preference relation holds for this pair ($a \succsim^P b$) if and only if a is at least as good as b for at least one compatible preference model.

Although UTA$^{\text{GMS}}$ [23] is the first method applying the ROR concepts, in the following, we shall describe the GRIP method [24] being its generalization taking into account intensity of preference. Then, we shall mention the other applications of the ROR that have been published later in several papers.

GRIP

In the UTA$^{\text{GMS}}$ method [23], which initiated the stream of further developments in ROR, the ranking of reference alternatives does not need to be complete as in the original UTA method [16]. Instead, the DM may provide pairwise comparisons just for those reference alternatives, he or she really wants to compare. Precisely, the DM is expected to provide a partial preorder $\succsim$ on A^R.

Obviously, one may also refer to the relations of strict preference $\succ$ or indifference $\sim$, which are defined as, respectively, the asymmetric and symmetric part of $\succsim$. The transition from a reference preorder to a value function is done in the following way: for $a^*, b^* \in A^R$,

$$\left.\begin{array}{l} U(a^*) \geq U(b^*) + \varepsilon, \text{ if } a^* \succ b^*, \\ U(a^*) = U(b^*), \text{ if } a^* \sim b^*, \end{array}\right\} E_1$$

where ε is a (generally small) positive value.

In some decision-making situations, the DMs are willing to provide more information than a partial preorder on a set of reference alternatives, such as "a^* is preferred to b^* at least as much as c^* is preferred to d^*." The information related to the intensity of preference is accounted by the GRIP method [24]. It may refer to the comprehensive comparison of pairs of reference alternatives on all criteria or to a particular criterion only. Precisely, in the holistic case, the DM may provide a partial preorder $\succsim^*$ on $A^R \times A^R$, whose meaning is: for $a^*, b^*, c^*, d^* \in A^R$,

$(a^*, b^*) \succsim^* (c^*, d^*) \iff a^*$ is preferred to b^* at least as much as c^* is preferred to d^*.

When referring to a particular criterion $g_j \in G$, rather than to all criteria jointly, the meaning of the expected partial preorder $\succsim_j^*$ on $A^R \times A^R$ is the following: for $a^*, b^*, c^*, d^* \in A^R$,

$(a^*, b^*) \succsim_j^* (c^*, d^*) \iff a^*$ is preferred to b^* at least as much as c^* is preferred to d^* on criterion g_j.

In both cases, the DM is allowed to refer to the strict preference and indifference relations rather than to weak preference only. The transition from the partial preorder expressing intensity of preference to a value function is the following: for $a^*, b^*, c^*, d^* \in A^R$,

$$\left.\begin{array}{l} U(a^*) - U(b^*) \geq U(c^*) - U(d^*) + \varepsilon, \text{ if } (a^*, b^*) \succ (c^*, d^*), \\ U(a^*) - U(b^*) = U(c^*) - U(d^*), \text{ if } (a^*, b^*) \sim (c^*, d^*), \\ u_j(a^*) - u_j(b^*) \geq u_j(c^*) - u_j(d^*) + \varepsilon, \text{ if } (a^*, b^*) \succ_j (c^*, d^*) \text{ for } g_j \in G, \\ u_j(a^*) - u_j(b^*) = u_j(c^*) - u_j(d^*), \text{ if } (a^*, b^*) \sim_j (c^*, d^*) \text{ for } g_j \in G. \end{array}\right\} E_2$$

In order to check if there exists at least one model compatible with the preferences of the DM, we solve the following linear programming problem:

$$\begin{array}{c} \varepsilon^* = \max \varepsilon, \quad \text{s.t.} \\ E \cup E_1 \cup E_2 = E^{\mathrm{DM}} \end{array}$$

If the set of constraints E^{DM} is feasible and $\varepsilon^* > 0$, then there exists at least one additive value function compatible with the preference information provided by the DM, otherwise no additive value function is compatible with the provided information. In this case, the analyst can decide to check for the cause of the incompatibility [25] or can continue the decision-aiding process accepting the incompatibility.

Denoting by $\mathcal{U}_{A^R}$ the set of value functions compatible with the preference information provided by the DM, in the GRIP method, three types of possible and necessary preference relations can be defined:

- $a \succsim^N b$ iff $U(a) \geq U(b)$ for all $U \in \mathcal{U}_{A^R}$, with $a, b \in A$,
- $a \succsim^P b$ iff $U(a) \geq U(b)$ for at least one $U \in \mathcal{U}_{A^R}$, with $a, b \in A$,
- $(a, b) \succsim^{*N} (c, d)$ iff $U(a) - U(b) \geq U(c) - U(d)$ for all $U \in \mathcal{U}_{A^R}$, with $a, b, c, d \in A$,

- $(a,b) \succsim^{*P} (c,d)$ iff $U(a) - U(b) \geq U(c) - U(d)$ for at least one $U \in \mathcal{U}_{AR}$, with $a,b \in A$,
- $(a,b) \succsim_j^{*N} (c,d)$ iff $u_j(a) - u_j(b) \geq u_j(c) - u_j(d)$ for all $U \in \mathcal{U}_{AR}$, with $a,b,c,d \in A$, $g_j \in G$,
- $(a,b) \succsim_j^{*P} (c,d)$ iff $u_j(a) - u_j(b) \geq u_j(c) - u_j(d)$ for at least one $U \in \mathcal{U}_{AR}$, with $a,b \in A$, $g_j \in G$.

Given alternatives $a,b,c,d \in A$, and the sets of constraints

$$\left.\begin{array}{l} U(b) \geq U(a) + \varepsilon \\ E^{\mathrm{DM}} \end{array}\right\} E^N(a,b), \qquad \left.\begin{array}{l} U(a) \geq U(b) \\ E^{\mathrm{DM}} \end{array}\right\} E^P(a,b)$$

$$\left.\begin{array}{l} U(c) - U(d) \geq U(a) - U(b) + \varepsilon \\ E^{\mathrm{DM}} \end{array}\right\} E^N(a,b,c,d), \qquad \left.\begin{array}{l} U(a) - U(b) \geq U(c) - U(d) \\ E^{\mathrm{DM}} \end{array}\right\} E^P(a,b,c,d)$$

$$\left.\begin{array}{l} u_j(c) - u_j(d) \geq u_j(a) - u_j(b) + \varepsilon \\ E^{\mathrm{DM}} \end{array}\right\} E_j^N(a,b,c,d), \qquad \left.\begin{array}{l} u_j(a) - u_j(b) \geq u_j(c) - u_j(d) \\ E^{\mathrm{DM}} \end{array}\right\} E_j^P(a,b,c,d)$$

we get that:

- $a \succsim^N b$ iff $E^N(a,b)$ is infeasible or it is feasible and $\varepsilon^N(a,b) \leq 0$, where $\varepsilon^N(a,b) = \max \varepsilon$, s.t. $E^N(a,b)$;
- $a \succsim^P b$ iff $E^P(a,b)$ is feasible and $\varepsilon^P(a,b) > 0$, where $\varepsilon^P(a,b) = \max \varepsilon$, s.t. $E^P(a,b)$;
- $(a,b) \succsim^{*N} (c,d)$ iff $E^N(a,b,c,d)$ is infeasible or it is feasible and $\varepsilon^N(a,b,c,d) \leq 0$, where $\varepsilon^N(a,b,c,d) = \max \varepsilon$, s.t. $E^N(a,b,c,d)$;
- $(a,b) \succsim^{*P} (c,d)$ iff $E^P(a,b,c,d)$ is feasible and $\varepsilon^P(a,b,c,d) > 0$, where $\varepsilon^P(a,b,c,d) = \max \varepsilon$, s.t. $E^P(a,b,c,d)$;
- $(a,b) \succsim_j^{*N} (c,d)$ iff $E_j^N(a,b,c,d)$ is infeasible or it is feasible and $\varepsilon_j^N(a,b,c,d) \leq 0$, where $\varepsilon_j^N(a,b,c,d) = \max \varepsilon$, s.t. $E_j^N(a,b,c,d)$;
- $(a,b) \succsim_j^{*P} (c,d)$ iff $E_j^P(a,b,c,d)$ is feasible and $\varepsilon_j^P(a,b,c,d) > 0$, where $\varepsilon_j^P(a,b,c,d) = \max \varepsilon$, s.t. $E_j^P(a,b,c,d)$;

As to properties of $\succsim^N$ and $\succsim^P$ on A, let us remind after [23] that

- $\succsim^N$ is a partial preorder on A,
- $\succsim^N \subseteq \succsim^P$,
- $a \succsim^N b$ and $b \succsim^P c \;\Rightarrow\; a \succsim^P c$, $\forall a,b,c \in A$,
- $a \succsim^P b$ and $b \succsim^N c \;\Rightarrow\; a \succsim^P c$, $\forall a,b,c \in A$,
- $a \succsim^N b$ or $b \succsim^P a$, $\forall a,b \in A$.

The above-mentioned properties are the minimal ones characterizing $\succsim^N$ and $\succsim^P$ [26]. Other interesting properties of $\succsim^N$ and $\succsim^P$ are the following [23]:

- $\succsim^P$ is strongly complete and negatively transitive,
- $\succ^P$ is complete, irreflexive, and transitive.

FURTHER DEVELOPMENTS

When looking at the final ranking, the DM is mainly interested in the position that is attained by a given alternative and, possibly, in its comprehensive score. Therefore, in the RUTA method [27], the kind of preference information that may be supplied by the DM have been extended by information referring to the desired rank of reference alternatives, that is, final positions and/or scores of these alternatives. Indeed, people are used to refer to the desired ranks of the alternatives in their judgments. In many real-world decision situations (e.g., evaluation of candidates for a certain position), they use statements such as a^* should be among the 5% of best/worst alternatives or b^* should be ranked in the second ten of alternatives. These statements refer to the range of allowed ranks that a particular alternative should attain. When using such expressions, people do not confront "one vs" as in pairwise comparisons or "pair vs" as in statements concerning intensity of preference, but rather rate a given alternative individually, comparing it with all the remaining alternatives jointly. Extreme ranking analysis [28] provides the worst and the best positions that each alternative can obtain considering the whole set $\mathcal{U}_{AR}$ of compatible value functions. See Refs 29–31 for some recent applications of the extreme ranking analysis methodology.

A great majority of methods designed to support the multiple criteria decision process, assuming that all evaluation criteria are considered at the same level. However, practical applications are often imposing a hierarchical structure of criteria. For example, in economic ranking, alternatives may be evaluated on indicators that aggregate evaluations on several subindicators, and these subindicators may aggregate another set of subindicators, etc. In this case, the marginal value functions may refer to all levels of the hierarchy, representing the values of particular scores of the alternatives on indicators, subindicators, sub-sub-indicators, etc. In multiple criteria hierarchy process (MCHP)[32–34], the DM may provide a partial preorder of reference alternatives in each node of the hierarchy and obtain a necessary and a possible preference relation on the whole set of alternatives in each node. MCHP permits to decompose a complex decision problem into a series of simpler subproblems.

UTADIS^{GMS} method [35] applies the ROR to sorting problems. The DM can provide preference information in terms of assignment of alternatives to intervals of classes obtaining as result, for each alternative, intervals of classes to which it can be assigned necessarily and possibly considering the whole set of compatible value functions. This approach has been subsequently extended to the DIS-CARD method, which additionally admits specification of desired class cardinalities [36].

ROR has been applied to extend the two best known families of outranking methods, which are ELECTRE [37] and PROMETHEE [28], giving birth to the ELECTRE^{GKMS} and the PROMETHEE^{GKS} methods. In these methods, the DM can provide preference information in terms of outranking of an alternative over another and/or can give comparisons among weights, possible intervals of weights, and intervals of thresholds. As a result, ROR gives necessary and possible outranking relations between alternatives considering the whole family of sets of preference model parameters (weights, indifference, preference and veto thresholds, and concordance cutting level) compatible with preference information.

Even if the additive model is among the most popular ones, some critics have been addressed to this model because it has to obey an often unrealistic hypothesis about preferential independence among criteria. In consequence, it is not able to represent *interactions* among criteria [38]. For example, consider evaluation of cars using such criteria as maximum speed, acceleration, and price. In this case, there may exist a negative interaction (*redundancy*) between maximum speed and acceleration because a car with a high maximum speed also has a good acceleration, so, even if each of these two criteria is very important for a DM who likes sport cars, their joint impact on reinforcement of preference of a more speedy and better accelerating car over a less speedy and worse accelerating car will be smaller than a simple addition of the two impacts corresponding to each of the two criteria considered separately in validation of this preference relation. In the same decision problem, there may exist a positive interaction (*synergy*) between maximum speed and price because a car with a high maximum speed usually also has a high price, and thus a car with a high maximum speed and relatively low price is very much appreciated. Thus, the comprehensive impact of these two criteria on the strength of preference of a more speedy and cheaper car over a less speedy and more expensive car is greater than the impact of the two criteria considered separately in validation of this preference relation.

To handle the interactions among criteria, one can consider *non-additive integrals*, such as Choquet integral and Sugeno integral (see, e.g., Ref. 39). ROR has been applied to such types of preference models originating the *NonAdditive Robust Ordinal Regression* (NAROR) [40]. However, the non-addtive integrals suffer from limitations within MCDA [41]; in particular, they need that the evaluations on all criteria are expressed on the same scale. This means that in order to apply a nonadditive integral it is necessary, for example, to estimate if the maximum speed of 200 km/h is as valuable as the price of €35, 000. Thus, in order to consider positive and negative synergies, UTA$^{\mathrm{GMS}}$ method has been extended to the UTA$^{\mathrm{GMS}}$-INT method [42], which considers a value function, composed not only of the sum of marginal nondecreasing value functions but also of penalty and bonus functions representing negative and positive synergies between criteria, respectively. On the basis of a value function similar to that one used in UTA$^{\mathrm{GMS}}$-INT, the customer satisfaction method MUSA$^{\mathrm{INT}}$ [43] applies the ROR concept to compare different customer profiles considering all the value functions compatible with the customers' preferences.

ROR has also been adapted to aid a group of DMs, $\mathcal{D} = \{d_1, \ldots, d_p\}$, to cooperate in view of taking a collective decision. In UTA$^{\mathrm{GMS}}$-GROUP and UTADIS$^{\mathrm{GMS}}$-GROUP methods [44], the collective results account for the preferences expressed by each DM. For example, based on the necessary ($\succsim^N_{d_r}$) and possible ($\succsim^P_{d_r}$) preference relations for each DM d_r, in the UTA$^{\mathrm{GMS}}$-GROUP method, four relations can be defined:

- $a \succsim^{N,N}_{\mathcal{D}} b : a \succsim^N_{d_r} b$ for all $d_r \in \mathcal{D}$,
- $a \succsim^{N,P}_{\mathcal{D}} b : a \succsim^N_{d_r} b$ for at least one $d_r \in \mathcal{D}$,
- $a \succsim^{P,N}_{\mathcal{D}} b : a \succsim^P_{d_r} b$ for all $d_r \in \mathcal{D}$,
- $a \succsim^{P,P}_{\mathcal{D}} b : a \succsim^P_{d_r} b$ for at least one $d_r \in \mathcal{D}$.

Considering results of four different types permits to indicate what would happen always (for all compatible functions), sometimes (for at least one compatible function), or never (for none of the compatible functions) with respect to a subset or to the whole set of DMs. In this way, one can investigate spaces of consensus and disagreement between the DMs.

Even if the recommendations obtained using ROR are "more robust" than a recommendation made using an arbitrarily chosen compatible model, in some decision-making situations, a score is needed to be known for different alternatives; for this reason, some users would like to see the "representative" model among all the compatible ones. The motto underlying this proposal is "one for all, all for one." The representative value function represents all compatible value functions, which also do contribute to its definition. On the basis of the ROR concept, the representative model is the compatible model maximizing the difference of values between alternatives a and b for which a is necessarily preferred to b but b is not necessarily preferred to a and minimizing the difference of values between alternatives a and b for which neither a is necessarily preferred to b nor b is necessarily preferred to a. The representative model concept has been introduced for the first time in Ref. 45 and then applied to deal with ranking and choice problems [46, 47], outranking methods [48], sorting problems [49], and group decision making [50].

In order to explain the necessary and possible preference relations given by ROR in terms of conditions on evaluation criteria, [51] proposes to couple ROR with DRSA [52]. Applying DRSA to the necessary and possible preference supplied by ROR, we get decision rules stating, for example, that the preference, either necessary or possible, of alternative a over alternative b is explained by a strong preference on criterion g_{j_1} and at least mild preference on criterion g_{j_2}. In this case, the strong preference on criterion g_{j_1} and the at least mild preference on criterion g_{j_2} become the arguments suggested by the rule for the preference of a over b. In a learning perspective, decision rules supplied by DRSA can be the starting point for an interactive procedure for analyzing and constructing the DM's preferences. It enables the DM's understanding of the conditions for the suggested recommendation and provides useful information about the role of particular criteria or their subsets.

ROR has been applied in multiple objective optimization (MOO; for an exhaustive collection of surveys on MOO, see Ref. 53) in Refs 54 and 55. In the first paper, GRIP method is used for an interactive exploration of the Pareto optimal set of a MOO. In the latter paper, pairwise comparisons provided by the DM are used to systematically contract a cone formed by the directions of the isoquants of all compatible achievement scalarizing functions. This cone is focusing the DM's attention on a subregion of the nondominated set that better corresponds to his or her preferences.

It is also worth mentioning that ROR has been applied to guide evolutionary multiobjective optimization (EMO), that is, a procedure that approximates the Pareto front of a multiobjective optimization problem evolving an initial population of solutions through multiple generation by breeding and mutation, toward the set of solutions most preferred by the DM [56, 57].

Recent extensions of the ROR approach are presented in Refs 58 and 59 where the Stochastic multiobjective acceptability analysis (SMAA) [60] and the ROR are put together under a unified decision support framework.

CONCLUSION

In this article, we have presented the ROR being a family of multicriteria decision-aiding methods aiming at considering not only one compatible preference model but also the whole set of models compatible with some preference information provided by the DM in terms of pairwise comparison of some alternatives, intensities of preference, rank-related requirements, or statements concerning interaction between criteria. We have presented the GRIP method being the extension of the first ROR method UTA^{GMS}. We have also mentioned other applications of the ROR, including hierarchy of criteria (MCHP), sorting problems (UTADIS^{GMS}), outranking methods (ELECTRE^{GKMS} and PROMETHEE^{GKS}), non-additive integrals (NAROR), group decisions (UTA^{GMS}-GROUP and UTADIS^{GMS}-GROUP), and MOO.

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